

Exercise 94

A farmer finds that if she plants 75 trees per acre, each tree will yield 20 bushels of fruit. She estimates that for each additional tree planted per acre, the yield of each tree will decrease by 3 bushels. How many trees should she plant per acre to maximize her harvest?

Solution

Since the yield of each tree decreases at a constant rate for each additional tree planted, a linear function can be used to model the yield.

$$y = mx + b$$

m , the slope, is the rate of increase.

$$y = -3x + b$$

Use the fact that $y = 20$ when $x = 75$ to determine b .

$$20 = -3(75) + b$$

$$20 + 3(75) = b$$

$$b = 245$$

The number of bushels per tree in an acre is then given by

$$y = -3x + 245.$$

Multiply this function by the number of trees to get the number of bushels harvested per acre.

$$\begin{aligned} H &= xy \\ &= x(-3x + 245) \\ &= -3x^2 + 245x \\ &= -3 \left(x^2 - \frac{245}{3}x \right) \\ &= -3 \left[\left(x - \frac{245}{3} \right)^2 + \frac{245^2}{9} \right] \\ &= -3 \left[\left(x - \frac{245}{3} \right)^2 - \frac{245^2}{3} \right] \\ &= -3 \left[\left(x - \frac{245}{3} \right)^2 - \frac{245^2}{3} \right] \\ &= -3 \left(x - \frac{245}{3} \right)^2 + \frac{60\,025}{3} \end{aligned}$$

The maximum harvest is $H = \frac{60\,025}{3} \approx 20\,008$ bushels per acre, which occurs if there are $x = \frac{245}{3} \approx 81$ trees planted per acre.